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Momentum Correlations for Identical Fermions

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The effects of intensity interferometry (HBT), which are mainly reflected by the measured momentum correlations, have been fully discussed since 1950's. The analogous momentum correlations between identical fermions are however less argued. Besides the fermion statistics, the momentum correlations between identical fermions are also determined by some other aspects, such as coulomb interaction, spin interactions and resonances formation. In this paper, we discussed the factors that influence the momentum correlations between Λ's, especially spin interaction. It is also argued that the momentum correlations between Λ's are affected significantly by the yield ratio of Σ^0/Λ, due to the limitation of experimental measurement. The mixture of Σ^0's and Λs would significantly weaken the measured two-particle momentum correlations of Λ's.

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INTRODUCTION

Intensity interferometry(HBT), which was firstly proposed by Hanbury-Brown and Twiss[1] as a method of measuring the sizes of stars, was later extended to particle physics by Goldhaber *et al*[2] to extract the space-time structure of $p\bar{p}$ annihilations by studying the angular distribution of identical pion pairs in the collisions. HBT interferometry is now regarded as a useful tool to study the space-time structures of fireballs produced in heavy-ion collisions[3, 4]. The HBT interferometry is an effect of Bose-Einstein statistics, which leads to an enhancement of the identical boson pairs when their relative momenta is small. The effect is usually represented by the measured two-particle momentum correlations. It has been argued that some dynamical information of the expanding fireball created in heavy-ion collisions can be obtained by the correlations. For instance, the information on the anisotropic transverse flow of the fireball can be extracted by measuring the correlation functions as a function of the emission angle[5, 6, 7]. In fact, the 2-particle momentum correlations of a many-body system are determined not only by the quantum statistics, but also the interactions between the particles produced in the source and the final-state interactions. The pure quantum statistics effects can be acquired only after all these influences have been removed carefully. For example, due to the repulsive coulomb interactions, the momentum correlations between identical charged pions are suppressed to some extent for small relative momenta, and the expected great enhancement can show up only after the coulomb interactions are removed correctly[8].

Although the theory of HBT effects[3, 4, 9] has been fully discussed since 1950's, and a lot of experimental results have been aquired by relativistic heavy-ion collisions at CERN/SPS[10, 11] and RHIC[8, 12, 13, 14, 15, 16, 17], there are still a lot of open questions in this area. Both theorists and experimentists are embarrassed by the so-called "HBT puzzle"[18] that hydrodynamic calculations, which provide perfect results of anisophic flows, yield strong disagreement with HBT radii and predict lack of energy dependence of the HBT radii for a fixed k_T bin[19, 20]. Besides "HBT" puzzle, the analogous momentum correlations between identical fermions are less argued. The conclusion can be made intuitively that the identical fermion pairs should be suppressed for small relative momentum due to Fermi-Dirac statistics. But little knowledge has been obtained by now to make a definite conclusion. Different from Boson's correlations, the spin interactions play an important role in the two-particle momentum correlations between identical fermions. The abundance of protons produced in heavy-ion collisions guarantees the statistics for momentum correlations between them, and SPS did measure the correlations [11]. But the complicated interactions between protons and the strong coulomb interactions are always big issues, which lead to the difficult to draw a definite conclusion. C. Greiner and B. Müller[21] proposed the momentum correlations between Λ's and discussed the influences by the possible existing di-Λ(H particle)[22, 23]. WA97 measured the Λ-Λ correlations[10] in Pb-Pb collisions at 158A GeV/c, but poor statistics refused any expected conclusions.

The relativistic heavy-ion collisions at RHIC is a factory of strangeness, where quite a lot of Λ's are produced for central collisions. This gives us a chance to measure the momentum correlations between Λ's with

better statistics. The most important advantage of the analysis of Λ correlations is that the coulomb interactions, if exist, can be neglected. What's more, We hardly had any opportunity before to study the interactions between baryons with strangeness, the momentum correlations between Λ's may give us a first glimpse of the interactions of strange baryons, however rough it might be. The investigation of the momentum correlations between Λ's at RHIC is therefore interesting and valuable at least from this point of view.

In this paper, we discussed those important factors that influence the momentum correlation between Λ's , one of which is the spin interactions between Λ's. It is also argued that the measured momentum correlations between Λ's are affected significantly by the yield ratio of Σ^0/Λ due to the limitation of experimental measurement.

The outline of this paper is as follows. The momentum correlation functions are presented in section 2, and the effects of spin interactions are argued in section 3. In section 4 we discussed the effects of Σ^0/Λ ratio on the measured momentum correlations. Finally a brief summary is given in the last section.

THE 2-PARTICLE CORRELATION FUNCTION

The two-particle correlation function $C_2(\vec{p}_{rel})$ is usually defined as

$$\begin{aligned} C_2(\vec{p}_{rel}) &= \frac{P(\vec{p}_1,\vec{p}_2)}{P(\vec{p}_1)P(\vec{p}_2)} \equiv 1+R(\vec{p}_{rel}), \qquad (1) \\ P(\vec{p}_1,\vec{p}_2) &= \frac{1}{\sigma_{12}}\frac{d^6\sigma_{12}}{d^3\vec{p}_1 d^3\vec{p}_2}, \\ P(\vec{p}_i) &= \frac{1}{\sigma_1}\frac{d^3\sigma_i}{d^3\vec{p}_i},\ (i=1,2), \end{aligned}$$

where $\vec{p}_{rel} = \vec{p}_2 - \vec{p}_1$ is the relative momentum, $P(\vec{p}_i$ represents the probability of detecting a particle with momentum $\vec{p}_i$, and $P(\vec{p}_1,\vec{p}_2)$ that of detecting two particles that one with momentum $\vec{p}_1$ and the other with momentum $\vec{p}_2$. $R(\vec{p}_{rel})$ measures the difference between the two-particle cross section and the product of the inclusive single-particle cross section. Correlations between particles would lead $R(\vec{p}_{rel})$ to deviate from zero. If there are no correlations, $R(\vec{p}_{rel})$ will vanish. Therefore $R(\vec{p}_{rel})$ is sometimes also called correlation function. For thermalized expanding fireballs which emit identical particles, the correlation function C_2 should depend on the space size of the source at freeze-out.

For Λ-Λ correlations, we can approximately adopt the non-relativistic expression to calculate the correlation function $C_2(\vec{p}_{rel})$. Following the steps by Greiner and Müller[21], let $D(\vec{r},t;\vec{p})$ be the relative probability that a particle with momentum $\vec{p}$ freezes out at a place $\vec{r}$ at time t, and normalized on the differential cross section,

$$\int d^3\vec{r}dtD(\vec{r},t;\vec{p}) = \frac{1}{\sigma}\frac{d^3\sigma}{d^3\vec{p}}.$$

The two-particle differential cross section may then be approximated as

$$\begin{aligned} &\frac{1}{\sigma_{12}}\frac{d^6\sigma_{12}}{d^3\vec{p}_1 d^3\vec{p}_2} \\ &= \int_{-\infty}^{+\infty} dt_1 dt_2 \int d^3\vec{r}_1 d^3\vec{r}_2 D(\vec{r}_1,t_1;\vec{p}_1)D(\vec{r}_2,t_2;\vec{p}_2) \\ &\cdot(2\pi)^6|\Psi_{1,2}(\vec{r}_1^{\,\prime},\vec{r}_2;\vec{p}_1,\vec{p}_2)|^2, \qquad (2) \end{aligned}$$

where $\vec{r}_1^{\,\prime} = \vec{r}_1 + \vec{v}(t_2 - t_1)$, $\vec{v} = \frac{\vec{p}_1+\vec{p}_2}{2m} = \vec{P}_{cm}/2m$, and $\psi_{1,2}$ is the exact 2-particle wave function in the final channel. We parameterize the distribution function $D(\vec{r},t;\vec{p})$ by the following simple gaussian ansatz,

$$D(\vec{r},t;\vec{p}) = \frac{1}{\sigma}\frac{d^3\sigma}{d^3\vec{p}}\left(\frac{1}{\pi^{3/2}}\frac{1}{r_0^3}\exp(-r^2/r_0^2)\right)\delta(t-t_0), \quad (3)$$

where t_0 denotes the moment of freeze-out, and r_0 defines the spatial size of the fireball at freeze-out. The δ function indicates the ignorance of the formation time of particles. Inserting (3) in (2) and comparing with (1), we obtain

$$\begin{aligned} C_2(\vec{p}_{rel}) &= \frac{1}{\pi^3}\frac{1}{r_0^6}\int d^3\vec{r}_1 d^3\vec{r}_2 e^{-(r_1^2+r_2^2)/r_0^2} \\ &\cdot(2\pi\hbar)^6|\Psi_{1,2}(\vec{r}_1,\vec{r}_2;\vec{p}_1,\vec{p}_2)|^2. \qquad (4) \end{aligned}$$

EFFECTS OF SPIN WAVE FUNCTIONS

The key step to calculate the correlation function (4) is then to find the exact two-particle wave function, $\Psi_{1,2}$. For baryons with strangeness, the interactions between them is not yet clear, but the final-state interactions of strange particles can be ignored as a first step, and we can focus our attention on the effects of the underlying quantum statistics, and express $\Psi_{1,2}$ simply as the production of plane waves of the particles.

For bosons, the total wave function $\Psi_{1,2}$ is just the spatial wave function itself, which should be symmetric under particle exchanges. It is easy to show that

$$C_2^B(\vec{p}_{rel}) = 1 + \exp(-p_{rel}^2 r_0^2/2). \qquad (5)$$

The case becomes a little complicated for fermions, which demand the total wave function to be asymmetric under particle exchanges. The total wave is the product of spatial wave function and spin wave function, and other internal wave functions such as isospin wave function, if exist. For spin-$\frac{1}{2}$ particles, there are spin singlet state, χ_A, and triplet states, χ_S, for a 2-particle system. The former corresponds to spin asymmetric state, while

the latter spin symmetric state. We can thus form two kinds of asymmetric total wave functions fermions with no other internal space,

$$\psi^S(\vec{p}_1,\vec{r}_1;\vec{p}_2,\vec{r}_2) = \phi^+\chi_A,$$
$$\psi^T(\vec{p}_1,\vec{r}_1;\vec{p}_2,\vec{r}_2) = \phi^-\chi_S,$$

where ϕ^+ and ϕ^- are respectively normalized symmetric and asymmetric spatial wave functions. Therefore, the total wave function, $\Psi_{1,2}$, can be express as a mixed state of singlet ψ^S and triplet ψ^T,

$$\Psi_{1,2} = \alpha\psi^S + \beta\psi^T, |\alpha|^2 + |\beta|^2 = 1. \qquad (6)$$

The momentum correlation function of Λ's is then given by

$$C_2^\Lambda(\vec{p}_{rel}) = 1 - (|\beta|^2 - |\alpha|^2)\exp(-p_{rel}^2 r_0^2/2), \qquad (7)$$

which depends apparently on the probability of spin singlet and triplet stats. If we assume spin singlet state and each component of triplet state has the same probability, that is, $\alpha = 1/2$ and $\beta = \sqrt{3}/2$, the correlation function becomes

$$C_2^\Lambda(\vec{p}_{rel}) = 1 - \frac{1}{2}\exp(-p_{rel}^2 r_0^2/2). \qquad (8)$$

This is exactly the formula given by Greiner and Müller[21].

Unfortunately we have in fact little (if not blank) knowledge about the spin interactions between hyperons. It is hardly to determine the value of α and β. The overall investigation on the interactions between nucleons are made mostly via p-p collisions. Deuteron, the two-particle system of n-p, is the only boundary state of nucleon-nucleon. The spin of deuteron is $S = 1$, which indicates that the spins of neutron and proton intend to be in the same direction and form a triplet state. But we have no idea whether a Λ-Λ system prefers triplet or singlet state.

If $\beta = \alpha = 1/\sqrt{2}$, the correlation function R_Λ would vanish, which means no effects of momentum correlations exhibit. When $\beta > \alpha$, it gives $R_\Lambda < 0$ and Fermi-Dirac suppression would dominate. If $\beta < \alpha$, it gives a desperate result, $R_\Lambda > 0$, which means that Bose-Einstein enhancement shows up!

If $\beta = 1$, that is, the system is in pure triplet state, $R_\Lambda = -\exp(-p_{rel}^2 r_0^2/2)$, which represents a full Fermi-Dirac suppression for small relative momentum. On the contrary, if $\alpha = 1$, that is, the system is in pure singlet state, $R_\Lambda = \exp(-p_{rel}^2 r_0^2/2)$, which represents that the correlation behaviour of Λ's is totally in the same way as bosons.

THE EFFECTS OF Σ^0/Λ RATIO

The production of Σ's in heavy-ion collisions gives sensitive influence on the measured Λ-Λ correlation functions. Σ^0 has the same value quark components as Λ, but a little heavier. It decays to Λ and a low energy photon. With the mean life $\tau \sim 10^{-20}s$, which is far longer than the life of the fireballs created by heavy-ion collisions, nearly all Σ^0's decay long after the freeze-out of the fireballs, compared with the life of the fireballs. The decay length of Σ^0 is of the order 10^4fm, which is small enough to prevent us distinguishing experimentally the decay vertex of Σ^0 between primary vertex, we can therefore hardly determine whether a Λ, when reconstructed, is emitted directly from the fireball or from Σ^0 decay.

Suppose that N Λ's are reconstructed in one event, which are "all" selected to be emitted from the source. If the feed-down from multiply strange hyperons, notably Ξ^0 and Ξ^-, is neglected, these N Λ's mainly come from two kinds of contributions,"primordial" Λ's that are really originated directly from the fireball, and "decayed" Λ's that come from Σ^0 decays. Suppose that the yield ratio of $\Sigma^0/\Lambda = \gamma$, typically $\gamma \leq 1$. The number of "primordial" Λ's is then $n_{prim} = N/(1+\gamma)$, while the number of "decayed" Λ's is $n_{decay} = \gamma N/(1+\gamma)$. Since the "decayed" Λ's are nearly all produced chaotically long after freeze-out of the fireball, there should be no significant momentum correlations between them. There should also be no harm to declare that there are no momentum correlations between "decayed" Λ's and "primordial" Λ's.

The correlation function $C_2 = 1 + R$ is experimentally obtained by the pairs in same events divided by pairs in mixed events. The pairs obtained from the N Λ's can be separated into three parts, those between "primordial" Λ's, $N(\Lambda\Lambda)$, those between "decayed" Λ's, $N(\Sigma^0\Sigma^0)$ and those between "primordial" and "decayed" Λ's , $N(\Lambda\Sigma^0)$,

$$N(\Lambda\Lambda) = \frac{1}{2}\frac{N}{1+\gamma}\left(\frac{N}{1+\gamma} - 1\right),$$
$$N(\Sigma^0\Sigma^0) = \frac{1}{2}\frac{\gamma N}{1+\gamma}\left(\frac{\gamma N}{1+\gamma} - 1\right),$$
$$N(\Lambda\Sigma^0) = \frac{\gamma N^2}{(1+\gamma)^2}.$$

Totally, $N(\Lambda\Lambda) + N(\Sigma^0\Sigma^0) + N(\Lambda\Sigma^0) = \frac{1}{2}N(N-1) = N_{pair}^t$, where N_{pair}^t is the total pairs of the N Λ's.

According to the discussion above, there is no contribution to the correlation function for $N(\Sigma^0\Sigma^0)$ and $N(\Lambda\Sigma^0)$, that is, $C_2 = 1$ for these two kinds of pairs. The experimentally obtained correlation function is therefore written as

$$\begin{aligned} C_2^{exp} &= \frac{N(\Lambda\Lambda)}{N_{pair}^t}(1+R_\Lambda) + \frac{N(\Sigma^0\Sigma^0)}{N_{pair}^t} + \frac{N(\Lambda\Sigma^0)}{N_{pair}^t}, \\ &= 1 + \frac{N-(1+\gamma)}{(1+\gamma)^2(N-1)}R_\Lambda, \end{aligned} \qquad (9)$$

where R_Λ is the correlation function of pure Λ's. For $N \gg 1$, $C_2^{exp} \simeq 1 + \frac{1}{(1+\gamma)^2}R_\Lambda$. If $\gamma = 1$, and suppose that the probability of singlet and each component of triplet

is equal,

$$C_2^{exp} \ = \ 1 + \frac{1}{4} R_\Lambda = 1 - \frac{1}{8} \exp(-p_{rel}^2 r_0^2/2). \quad (10)$$

This means the the effects of pure Λ's momentum correlations would be significantly weakened if we can not identify those Λ's from Σ^0 decay. So we can hardly observe significant suppression of Λ-Λ correlations in heavy-ion collisions.

CONCLUSIONS

we investigated the momentum correlation between identical fermions, especially that between Λ's produced in heavy-ion collisions. Different from bosons, spin interactions play an important role in the behaviour of fermion's correlations. For Λ's, if the spin wave function of Λ-Λ system is symmetric, the asymmetric space wave function leads to a total suppression at small relative momentum. If the spin wave function is asymmetric, the symmetric space wave function, however, leads to a total enhancement at small relative momentum, just the same as the behaviour of identical bosons. The lack of knowledge on the spin interactions makes it complicated to draw a conclusion whether the correlation functions should suppress or enhance at small relative momentum. It is also hard to say to what extent the suppression or enhancement would be.

It is also argued that Σ^0/Λ yield ratio affects the measured momentum correlations of Λ. A high ratio would weaken the the correlation effects significantly. For $\Sigma^0/\Lambda = 1$, the measured effect of correlations would be reduced to only about one quarter of that of correlations between pure Λ's. In fact, the number of Λ's N that can be reconstructed for Au-Au central collisions at RHIC[24] is quite small. In this case, the deviation of C_2^{exp} from 1 is therefore even smaller.

The trouble is that neither the spin interactions between Λ's nor the Σ^0/Λ yield ratio is clear to us. If the spin interactions could be clearly understood, the behaviors of pure Λ's correlations would be determined to a great extent. The Σ^0/Λ ratio would therefore be extracted from the measured momentum correlation function. The Σ^0/Λ is important to to heavy-ion collision since it can give us information on the mechanisms of hadronization, e.g. recombination and fragmentation mechanisms[25, 26, 27]. On the other hand, if we could get a definite Σ^0/Λ yield ratio via some other approaches, the information on the interactions between Λ's could then be obtained after the influence of Σ^0/Λ was removed.

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[1] R. Hanbury-Brown and R. Q. Twiss, Nature **178**, 1046(1956)
[2] G. Goldhaber *et al*, Phys. Rev. **120**, 300(1960)
[3] U. Heinz and B. Jacak, Ann. Rev. Nucl. Part. Sci. **49**, 529(1999)
[4] S. Pratt, Phys. Rev. Lett. **53**, 1219(1984)
[5] S. A. Voloshin and W. E. Cleland, Phys. Rev. C**53**, 896(1996)
[6] S. A. Voloshin and W. E. Cleland, Phys. Rev. C**54**, 3212(1996)
[7] U. Heinz, A. Hummel, M. A. Lisa, U. A. Wiedemanm, Phys. Rev. C**66** 044903(2002)
[8] C. Adler *et al* (STAR Collaboration), Phys. Rev. Lett.**87**, 082301(2001)
[9] T. Csorto, nucl-th/0505019
[10] E. Andersen *et al*, J. Phys. G**25**, 423(1999)
[11] H. Appelshauser et al., Phys. Lett. B**467**, 21(1999)
[12] S. Johnson, Nucl. Phys. A**698**, 603(2002)
[13] A. Enokizono, Nucl. Phys. A**715**, 595(2003)
[14] J. Adams *et al* (STAR Collaboration), Phys. Rev. Lett. **91**, 262301(2003)
[15] J. Adams *et al* (STAR Collaboration), Phys. Rev. Lett. **91**, 262302(2003)
[16] J. Adams *et al* (STAR Collaboration), Phys. Rev. Lett. **93**, 012301(2004)
[17] J. Adams *et al* (STAR Collaboration), Phys. Rev. C**71**, 044906(2005)
[18] D. Magestro, J. Phys. G**31**, 265(2005)
[19] U. Heinz and P. Kolb, Nucl. Phys. A**702** 269(2002)
[20] P. Kolb and U. Heinz, nucl-th/0305084
[21] C. Greiner and B. Müller, Phys. Lett. B**219**, 199(1989)
[22] R. L. Jaffe, Phys. Rev. Lett. **38**, 195(1977)
[23] R. L. Jaffe, Phys. Rev. Lett. **38**, 1617(E)(1977)
[24] C. Adler *et al* (STAR Collaboration), Phys. Rev. Lett. **89**, 092301(2002)
[25] V. Greco and C. Ko, J. Phys. G**31**, s407(2005)
[26] V. Greco and C. Ko, nucl-th/0412043
[27] R. Fries J. Phys. G**31**, S379(2005)